Homework 11

P10.1.3 Determine $\mathbf{I_O}$ in Figure P10.1.3.

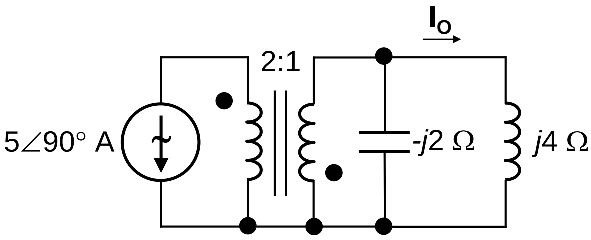


Figure P10.1.3

Solution: A current of *j*5 A leaves the marked terminal on the primary side. It follows that a positive current of *j*10 A leaves the unmarked terminal on the secondary side. From current division, $\mathbf{I_O} = j10(-j2/(-j2 + j4) = -j10$ A.

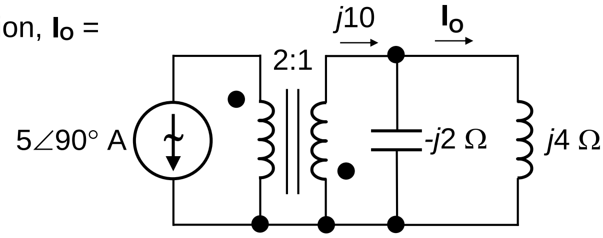


Figure P10.1.3-1

P10.1.11 Determine $\mathbf{V_O}$ in Figure P10.1.11.

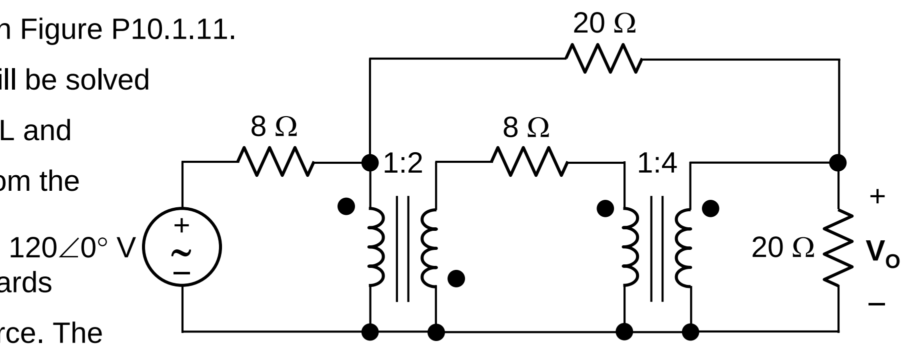


Figure P10.1.11

Solution: The problem will be solved by applying KVL and KCL starting from the output and working backwards toward the source. The current through the 20 Ω resistor at the output is $\mathbf{V_O}/20$ A. Denoting the secondary current of the transformer on the RHS by $\mathbf{I}$, the primary current is $4\mathbf{I}$, the current in the upper 20 Ω resistor is

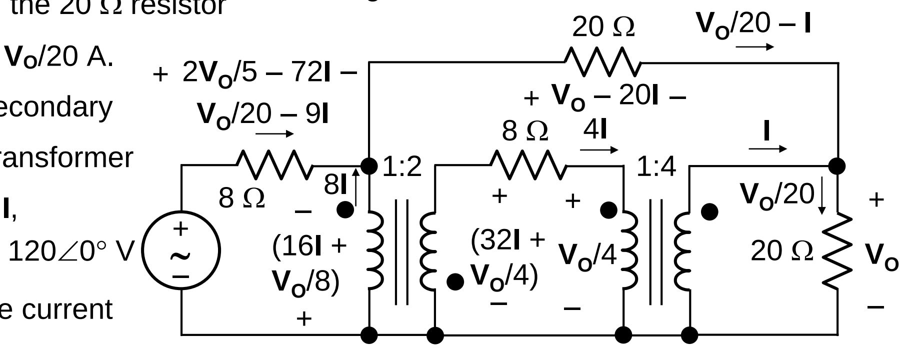


Figure P10.1.11-1

$(\mathbf{V_O}/20 - \mathbf{I})$, and the voltage across this resistor is $(\mathbf{V_O} - 20\mathbf{I})$. The secondary voltage of the transformer on the LHS is $(32\mathbf{I} + \mathbf{V_O}/4)$ of the polarity indicated. The primary voltage of this transformer is $(16\mathbf{I} + \mathbf{V_O}/8)$ and the primary current is $8\mathbf{I}$, both of the polarity indicated. The current in the 8 Ω resistor is $(\mathbf{V_O}/20 - 9\mathbf{I})$. From KVL around the mesh on the LHS is: $120 = (2\mathbf{V_O}/5 - 72\mathbf{I} - \mathbf{V_O}/8 - 16\mathbf{I}) = (11\mathbf{V_O}/40 - 88\mathbf{I})$. KVL around the primary voltage, the 20 Ω resistor and the output is: $(-16\mathbf{I} - \mathbf{V_O}/8 - \mathbf{V_O} + 20\mathbf{I} - \mathbf{V_O} = 0$, which gives $\mathbf{I} = 17\mathbf{V_O}/32$. Substituting for I gives $\mathbf{V_O} = -4800/1859 = 2.58\angle 180°$.

P10.1.20 Determine $\mathbf{V_L}$ in Figure P10.1.20.

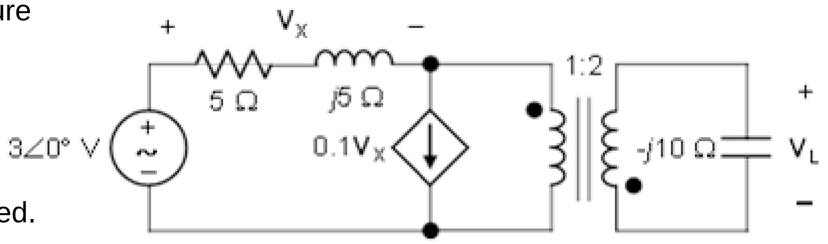


Figure P10.1.20

Solution: TEC as seen from the primary winding will be derived. It is seen that $\mathbf{V_X} = 0.1\mathbf{V_X}(5 + j5)$, which makes $\mathbf{V_X} = 0$ and $\mathbf{V_{Th}} = 3$ V. On short circuit, $\mathbf{V_X} = 3$ V, so that $0.1\mathbf{V_X} = 0.3$ V; from KCL, $3/(5 + j5) = 0.3 + \mathbf{I_{SC}}$, which $\mathbf{I_{SC}} = -j0.3$ A, so that $Z_{Th} = 3/(-j0.3) = j10\ \Omega$. TEC becomes as shown. Reflecting this to the secondary, it follows from voltage division that $\mathbf{V_L} = -(-j10/j30)\times 6 = 2$ V.

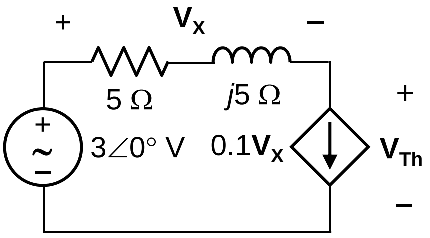


Figure P10.1.20-1

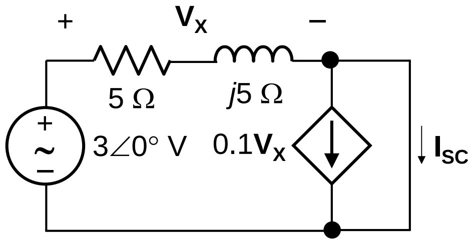


Figure P10.1.20-2

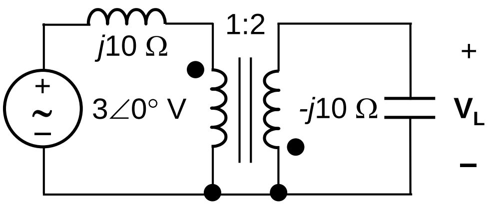


Figure P10.1.20-3

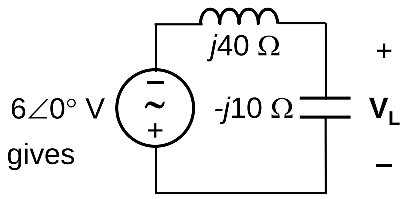


Figure P10.1.20-4

P10.1.30 Determine $\mathbf{I_{SRC}}$ in Figure P10.1.30.

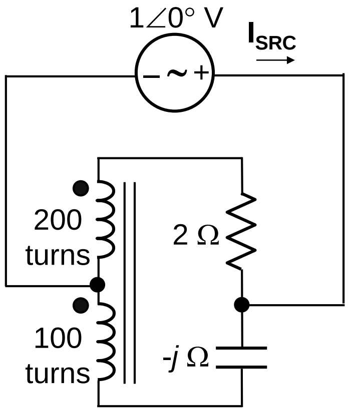


Figure P10.1.30

Solution: The voltages and currents of the ideal autotransformer may be assigned as shown. From KVL in the mesh involving *R*, *C*, and the autotransformer, $2\mathbf{I_2} + 2\mathbf{V_1} + \mathbf{V_1} - (-j2\mathbf{I_2}) = 0$, or $3\mathbf{V_1} = -2(1 + j)\mathbf{I_2}$. From KVL in the upper mesh, $2\mathbf{I_2} + 2\mathbf{V_1} = 1\angle 0°$. Substituting for $\mathbf{V_1}$, $2\mathbf{I_2} - (4/3)(1 + j)\mathbf{I_2} = 1\angle 0° = (2/3)(1 - j2)\mathbf{I_2}$, which gives, $\mathbf{I_2} = \dfrac{1.5}{1 - j2}$. Since $\mathbf{I_{SRC}} = 3\mathbf{I_2}$,

$$\mathbf{I_{SRC}} = \frac{4.5}{1 - j2} = 0.9(1 + j2) = 2.01\angle 63.4^\circ \text{ A.}$$

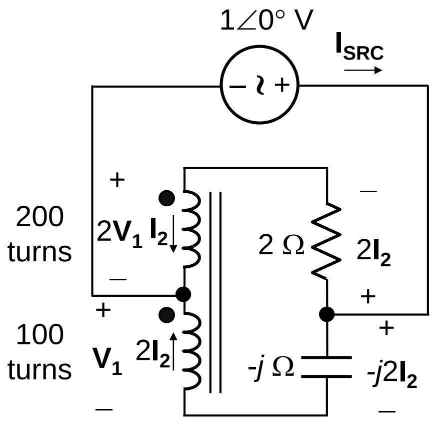


Figure P10.1.30-1

P10.1.36 Derive TEC looking into terminals 'ab' in Figure P10.1.36.

Solution: On open circuit, the equations for the transformer are:

$$\frac{\mathbf{V_2}}{\mathbf{V_1}} = \frac{300+200}{200} = 2.5\,; \mathbf{V_2} = 10 - 50\mathbf{I_2}$$

$$\mathbf{I_2} = \frac{V_1}{-j8} + \mathbf{I_1}\,;\ 300\mathbf{I_2} + 200\mathbf{I_1} = 0.$$

Substituting $\mathbf{V_1}$, $\mathbf{I_2} = \frac{\mathbf{V_2}}{-j8} + \mathbf{I_1}$.

Substituting $\mathbf{I_1}$, $\mathbf{I_2} = \frac{\mathbf{V_2}}{-j8} - \frac{\mathbf{3}}{\mathbf{2}}\mathbf{I_2}$, or $50\mathbf{I_2} = j\mathbf{V_2}$.

Substituting $50\mathbf{I_2}$, $\mathbf{V_2} = 10 - j\mathbf{V_2}$, or

$\mathbf{V_2} = \frac{10}{1+j} = 5(1-j)$ V, and $\mathbf{V_{Th}} = 2\mathbf{V_2} =$ $10(1 - j)$ V.

When a test source $\mathbf{V_T}$ is applied, with the independent source set t zero, $\mathbf{V_T} = 2\mathbf{V_2} = 100\mathbf{I_2}$. $\mathbf{V_1} = \mathbf{V_T}/5 = -j8\mathbf{I_1}$, or $\mathbf{I_1} = j\mathbf{V_T}/40$. The mmf equation is: $500\mathbf{I_T} + 300(\mathbf{I_T} - \mathbf{I_2}) + 200(\mathbf{I_T} - \mathbf{I_2} - \mathbf{I_1}) = 0$, or $10\mathbf{I_T} - 5\mathbf{I_2} - 2\mathbf{I_1} = 0$, Substituting $\mathbf{I_1}$ and $\mathbf{I_2}$ in terms of $\mathbf{V_T}$ gives:

$$\frac{\mathbf{V_T}}{\mathbf{I_T}} = \frac{200}{1+j} = 100(1-j)\ \Omega = Z_{Th}.$$

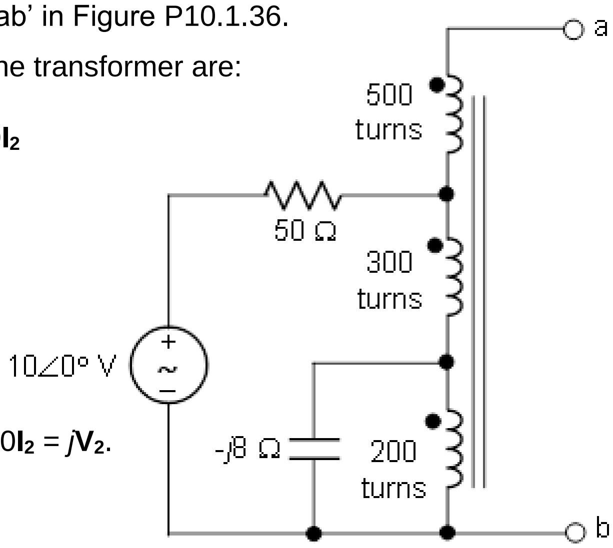


Figure P10.1.36

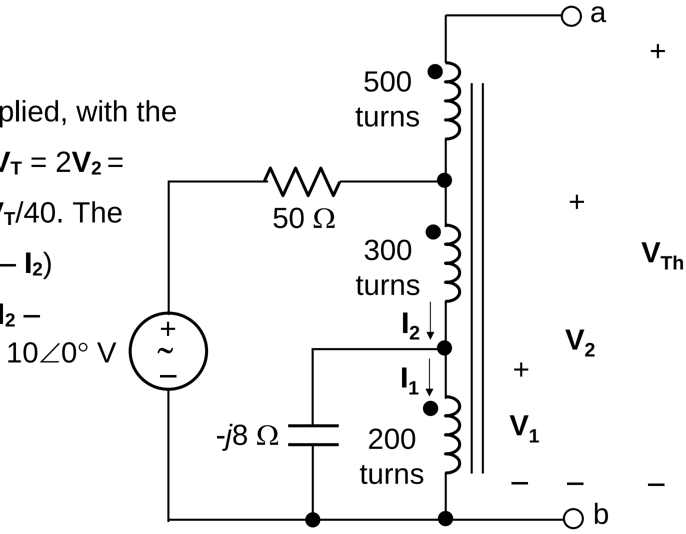


Figure P10.1.36-1

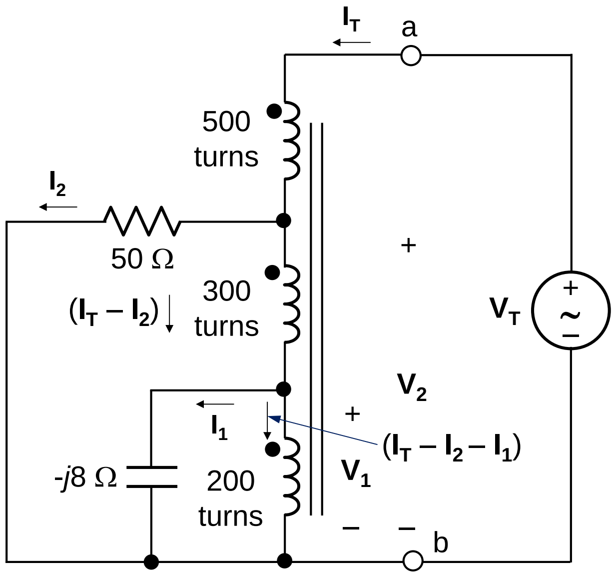


Figure P10.1.36-2

P10.1.40 Derive TEC looking into terminal 'ab' in Figure P10.1.40.

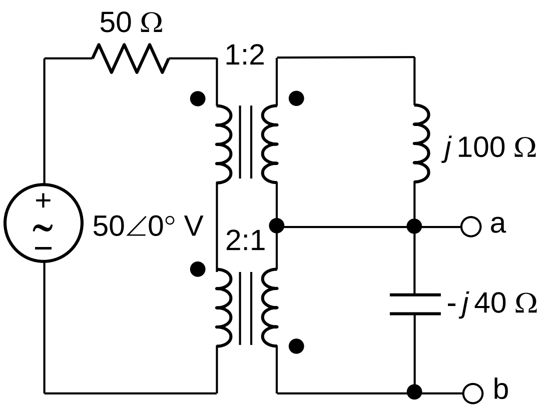
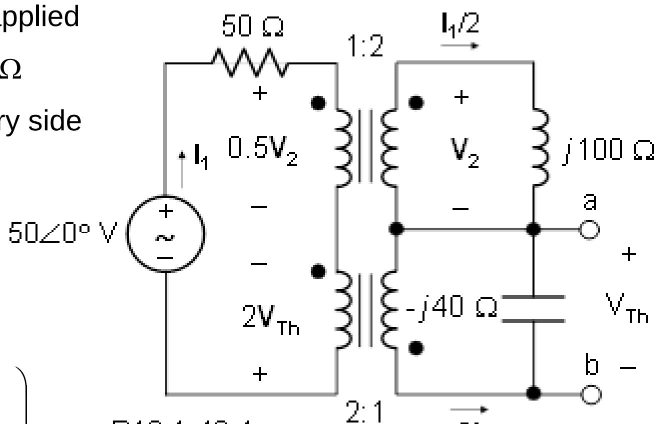


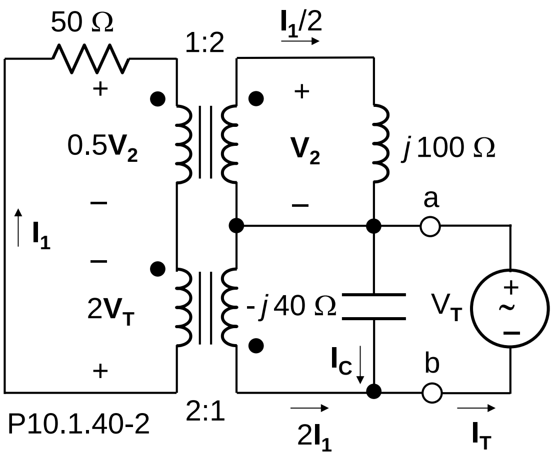
Figure P10.1.40

Solution: $\mathbf{V_2} = j50\mathbf{I_1}$, $\mathbf{V_{Th}} = -j40(-2\mathbf{I_1})$, $50 = 50\mathbf{I_1} + 0.5\mathbf{V_2} - 2\mathbf{V_{Th}}$. Substituting for $\mathbf{I_1}$ and $\mathbf{V_2}$ in terms of $\mathbf{V_{Th}}$ gives: $50 = -j\frac{5}{8}\mathbf{V_{Th}} + \frac{5}{16}\mathbf{V_{Th}} - 2\mathbf{V_{Th}}$; or, $50 = -\left(\frac{27}{16} + j\frac{5}{8}\right)\mathbf{V_{Th}}$, or, $\mathbf{V_{Th}} = -26.1 + j9.65$ V.

Let a test voltage source be applied between terminals 'ab'. The $j100\ \Omega$ impedance reflected to the primary side is $j25\ \Omega$. $\mathbf{I_1} = \frac{2\mathbf{V_T}}{50 + j25}$; $\mathbf{I_T} = \mathbf{I_C} + 2\mathbf{I_1} = \mathbf{V_T}\left(\frac{4}{50 + j25} - \frac{1}{j40}\right)$. It follows that $\frac{\mathbf{V_T}}{\mathbf{I_T}} = \left(\frac{1}{\frac{4}{50 + j25} - \frac{1}{j40}}\right) = \left(\frac{j40(50 + j25)}{-50 + j135}\right) = 15.44 + j1.69\ \Omega$.



P10.1.40-1



P10.1.40-2

P10.2.5 Determine L_{eq} in Figure P10.2.5.

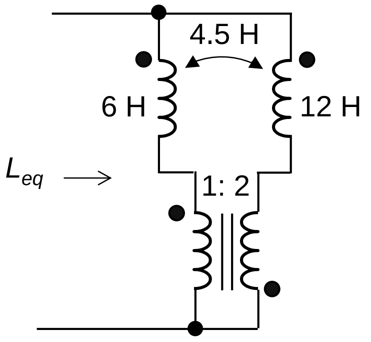


Figure P10.2.5

Solution: It is seen from the current relations for the ideal transformer that $\mathbf{I}/2 = (\mathbf{I_T} - \mathbf{I})$, or $\mathbf{I} = 2\mathbf{I_T}/3$. From KVL for each of the two branches:

$j\omega 6\mathbf{I} + j\omega 4.5(\mathbf{I_T} - \mathbf{I}) + \mathbf{V} = \mathbf{V_T}$ and

$j\omega 12(\mathbf{I_T} - \mathbf{I}) + j\omega 4.5\mathbf{I} - 2\mathbf{V} = \mathbf{V_T}$

Eliminating **V** and **I** between these equations

gives $j\omega 6\mathbf{I_T} = \mathbf{V_T}$, so that $L_{eq} = \mathbf{V_T}/j\omega\mathbf{I} = 6$ H.

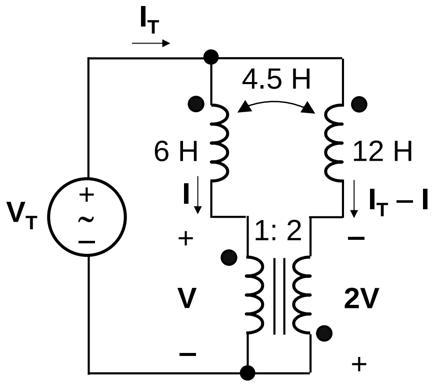


Figure P10.2.5-1

P10.2.7 Determine a in Figure P10.2.7 so that $Y_{in} = 0$, assuming $\omega = 1$ Mrad/s

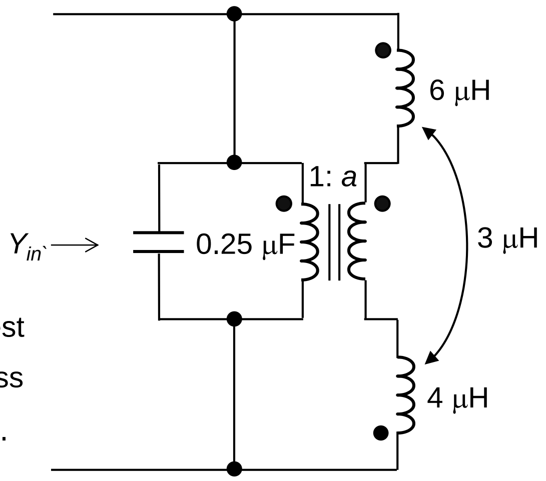


Figure P10.2.7

Solution: The two coupled coils have L_{eq} = $6 + 4 - 2\times3 = 4$ μH, and an impedance of $j\omega L_{eq} = j4\ \Omega$; $1/j\omega C = 1/(j10^6\times0.25\times10^{-6})$ = $-j4\ \Omega$.

When a test source $\mathbf{V_T}$ is applied, the test current $\mathbf{I_T}$ should be zero. The voltage across $j\omega L_{eq}$ is $(\mathbf{V_T} - a\mathbf{V_T})$, so that $\mathbf{I} = (1 - a)\mathbf{V_T}/(j4)$. From KCL at the upper node, $a\mathbf{I} = \mathbf{I} + \mathbf{V_T}/(-j4)$, or $\mathbf{I} = \mathbf{V_T}/[(j4)(1 - a)]$. Equating the two expressions of $\mathbf{I}$: $\dfrac{1-a}{j4} = \dfrac{1}{j4(1-a)}$, or,

$(1-a)^2 = 1$, $a = 1 \pm 1$, or $a = 2$.

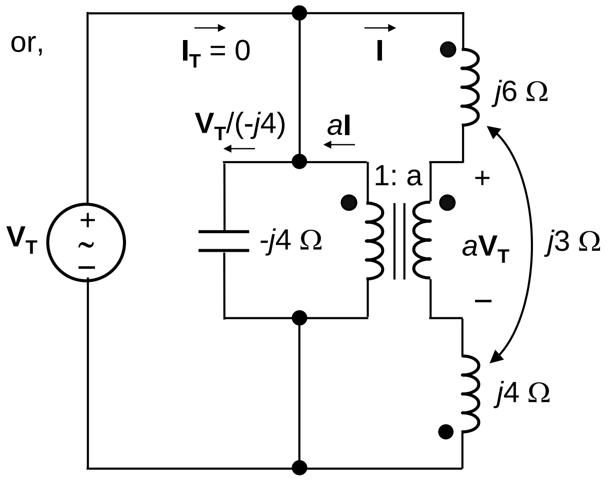


Figure P10.2.7-1